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Adjoined Magnetic PDE in openCFS

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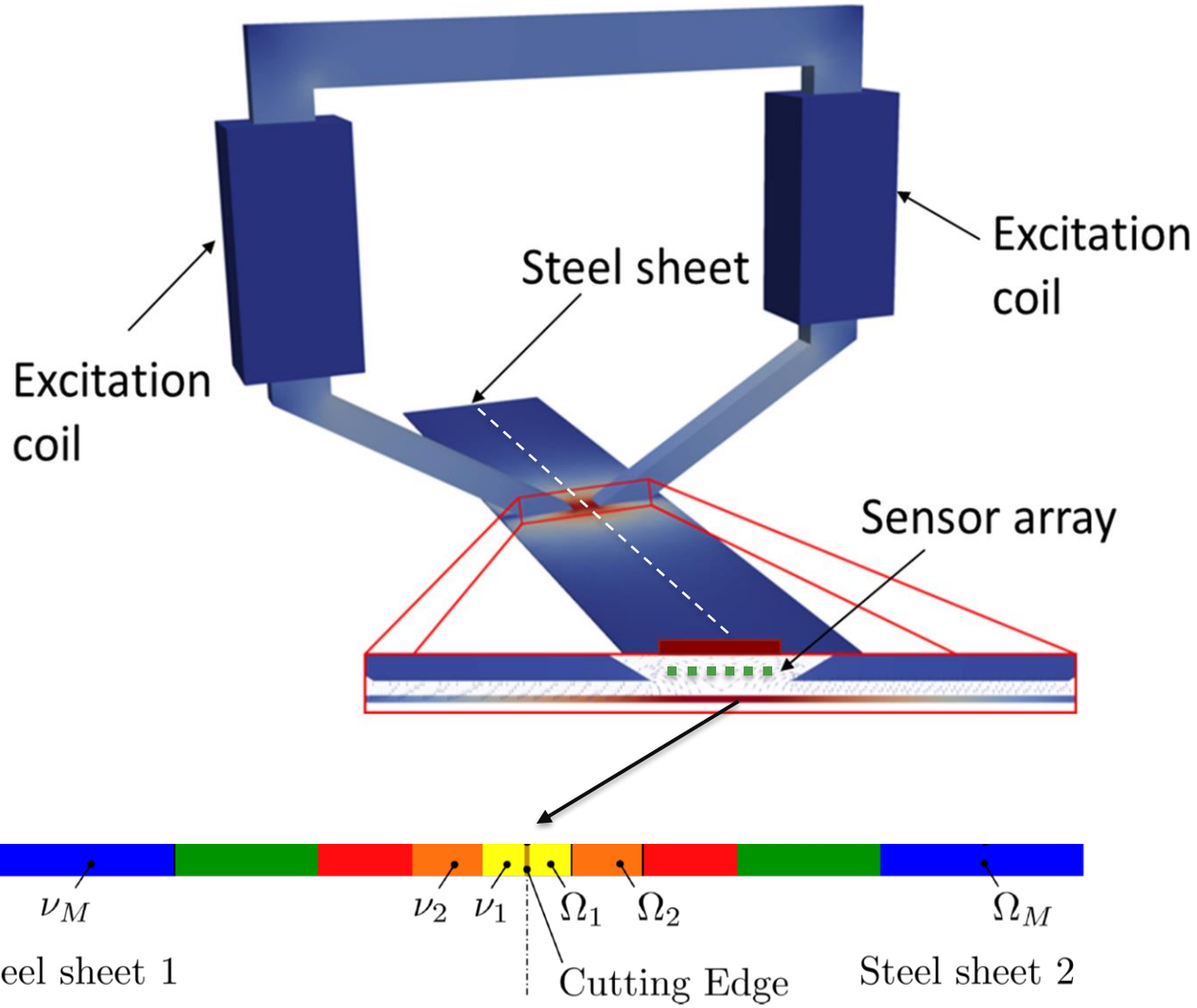
Lab Measurement Setup

Determine the actual degradation of electrical steel sheets due to cutting, punching, etc.



Magnetostatics:

$$\nabla \times \nu(\boldsymbol{B}) \nabla \times \boldsymbol{u} = \boldsymbol{g}\,; \qquad \boldsymbol{B} = \nabla \times \boldsymbol{u}$$

$\boldsymbol{B}$	...	magnetic flux density
$\boldsymbol{u}$	...	magnetic vector potential
ν	...	magnetic reluctivity
$\boldsymbol{g}$	...	current density

Formulation for parameter identification

Solve

$$\min_{\boldsymbol{u}\in V,\,\boldsymbol{\theta}\in\mathbb{R}^{n_\theta}} J(\boldsymbol{u},\boldsymbol{\theta}) \quad \text{s.t.} \quad \mathcal{A}(\boldsymbol{u},\boldsymbol{\theta}) = \boldsymbol{\nabla}\times\nu(\boldsymbol{u},\boldsymbol{\theta})\boldsymbol{\nabla}\times\boldsymbol{u} - \boldsymbol{g} = \boldsymbol{0}$$

with

$$J(\boldsymbol{u},\boldsymbol{\theta}) = \frac{1}{2}\sum_{p=1}^{N_\mathrm{p}}\sum_{e=1}^{N_\mathrm{e}}\sum_{s=1}^{N_\mathrm{s}} \frac{1}{|\Omega_s^{\mathrm{sens}}|}\int_{\Omega_s^{\mathrm{sens}}} \left|\,\boldsymbol{B}_{p,e,s} - \boldsymbol{B}_{p,e,s}^{\mathrm{meas}}\,\right|^2 \mathrm{d}\Omega + \underbrace{\alpha(\boldsymbol{\theta}-\boldsymbol{\theta}^{\mathrm{ref}})^T\boldsymbol{D}(\boldsymbol{\theta}-\boldsymbol{\theta}^{\mathrm{ref}})}_{\text{regularization term}}$$

N_p	...	positions of sensor-actuator
N_s	...	number of sensors
N_e	...	number of excitation levels
$\boldsymbol{B}_{p,e,s}$	...	computed magnetic field at sensor s, for excitation level e and sensor-actuator position p
$\boldsymbol{B}_{p,e,s}^{\mathrm{meas}}$	...	measured magnetic field at sensor s, for excitation level e and sensor-actuator position p

α	...	regularization parameter
$\boldsymbol{\theta} = (\boldsymbol{\theta}_1,\ldots,\boldsymbol{\theta}_{N_\mathrm{sd}})^T$	...	vector of parameters
$\boldsymbol{\theta}^{\mathrm{ref}}$	...	vector of reference parameters
$\boldsymbol{D}$	...	diagonal weighting matrix

Formulation for parameter identification

Steps of the inverse method based on an optimizer using gradient information:

- Perform measurements with an appropriate setup
- Solve the forward equation (magnetostatic PDE)[1,2]
- Solve the adjoint equation
- Compute the gradients of the objective function w.r.t. the material model parameters
- Apply an efficient gradient based optimizer

[1]L. D. Domenig, K. Roppert and M. Kaltenbacher. ‘Incorporation of a 3-D Energy-Based Vector Hysteresis Model Into the Finite Element Method Using a Reduced Scalar Potential Formulation. IEEE Trans. Magn. (2024).

[2]H. Egger, F. Engertsberger, L. Domenig, K. Roppert and M. Kaltenbacher. ‘On nonlinear magnetic field solvers using local Quasi-Newton updates’. Comput. Math. Appl. (2025).

Formulation for parameter identification

Since $\mathcal{A}$ is self adjoint, the left hand side of the adjoint equation in its weak formulations reads as

$$\int_\Omega \nu(B_{p,e})\ \boldsymbol{\nabla}\times\boldsymbol{w}_{p,e}\cdot\boldsymbol{\nabla}\times\boldsymbol{v}\ \mathrm{d}\Omega+\int_\Omega \nu'(B_{p,e})\ \boldsymbol{\nabla}\times\boldsymbol{w}_{p,e}\cdot\boldsymbol{e}_{B_{p,e}}\ B_{p,e}\ \boldsymbol{e}_{B_{p,e}}\cdot\boldsymbol{\nabla}\times\boldsymbol{v}\ \mathrm{d}\Omega$$

with

$B_{p,e}$	...	computed magnetic field (absolute value) by the forward equation (magnetostatic) for excitation level e and sensor-actuator position p
$\boldsymbol{w}$	..	adjoint unknown
$\boldsymbol{v}$	..	test function for the adjoint PDE
$\boldsymbol{e}_{B_{p,e}} = \boldsymbol{B}_{p,e}/B_{p,e}$	..	unit vector of the magnetic field (forward solution)

Please note that the left hand side of the adjoint PDE in the weak form is the same as for the forward PDE in the weak sense

This means for the computation that one just uses the system matrix of the forward problem

Formulation for parameter identification

For the right hand side, we explore the Gâteaux - derivative and obtain for each position *p* and excitation level *e*

$$\lim_{\varepsilon \to 0} \frac{1}{\varepsilon} \Big(J(\boldsymbol{u} + \varepsilon \boldsymbol{v}, \boldsymbol{\theta}) - J(\boldsymbol{u}, \boldsymbol{\theta}) \Big) = - \sum_{s=1}^{N_\mathrm{s}} \frac{1}{|\Omega_s^{\mathrm{sens}}|} \int_{\Omega_s^{\mathrm{sens}}} \left(\boldsymbol{B}_{p,e,s}^{\mathrm{meas}} - \boldsymbol{B}_{p,e,s} \right) \cdot \boldsymbol{\nabla} \times \boldsymbol{v} \, \mathrm{d}\Omega$$

N_s	...	number of sensors
$\boldsymbol{B}_{p,e,s}$	...	computed magnetic field at sensor s, for excitation level e and sensor-actuator position p
$\boldsymbol{B}_{p,e,s}^{\mathrm{meas}}$	...	measured magnetic field at sensor s, for excitation level e and sensor-actuator position p
$\boldsymbol{v}$	..	test function for the adjoint PDE

Please note: The adjoint equation is always linear!

Formulation for parameter identification

The functional is

$$J(\boldsymbol{u},\boldsymbol{\theta}) = \frac{1}{2}\sum_{p=1}^{N_\mathrm{p}}\sum_{e=1}^{N_\mathrm{e}}\sum_{s=1}^{N_\mathrm{s}}\frac{1}{|\Omega_s^{\mathrm{sens}}|}\int_{\Omega_s^{\mathrm{sens}}} \left|\, \boldsymbol{B}_{p,e,s} - \boldsymbol{B}_{p,e,s}^{\mathrm{meas}} \,\right|^2 \mathrm{d}\Omega + \underbrace{\alpha(\boldsymbol{\theta}-\boldsymbol{\theta}^{\mathrm{ref}})^T \boldsymbol{D}(\boldsymbol{\theta}-\boldsymbol{\theta}^{\mathrm{ref}})}_{\text{regularization term}}$$

- Gradient of reduced objective function

$$j(\boldsymbol{\theta}) = J(\boldsymbol{u},\boldsymbol{\theta}) + < \mathcal{A}(\boldsymbol{u},\boldsymbol{\theta}), \boldsymbol{w} > ; \qquad \frac{\partial j}{\partial \theta_i} = \frac{\partial J}{\partial \theta_i} + \left\langle \frac{\partial \mathcal{A}}{\partial \theta_i}, \boldsymbol{w} \right\rangle$$

- For a nonlinear material law (3 parameters per subdomain)

$$\nu_l(B) = c_{0,l} + c_{1,l} B^{c_{2,l}} \quad \text{on } \Omega_l\,; \quad l = 1,\ldots,N_{\mathrm{sd}}; \quad \boldsymbol{\theta} = \left(c_{0,l}\,, c_{1,l}\,, c_{2,l}\right)^T$$

we arrive at

$$\left\langle \frac{\partial \mathcal{A}}{\partial \theta_i}(\boldsymbol{u},\boldsymbol{\theta}), \boldsymbol{w} \right\rangle = \sum_{p=1}^{N_\mathrm{p}}\sum_{e=1}^{N_\mathrm{e}}\int_{\Omega} \frac{\partial \nu(B_{p,e})}{\partial \theta_i}\, \boldsymbol{\nabla}\times \boldsymbol{u}_{p,e} \cdot \boldsymbol{\nabla}\times \boldsymbol{w}_{p,e}\, \mathrm{d}\Omega'$$

Forward solution of vector potential — Adjoint solution

Formulation for parameter identification

The functional is

$$J(\boldsymbol{u},\boldsymbol{\theta}) = \frac{1}{2}\sum_{p=1}^{N_\mathrm{p}}\sum_{e=1}^{N_\mathrm{e}}\sum_{s=1}^{N_\mathrm{s}}\frac{1}{|\Omega_s^{\mathrm{sens}}|}\int_{\Omega_s^{\mathrm{sens}}} \left|\, \boldsymbol{B}_{p,e,s} - \boldsymbol{B}_{p,e,s}^{\mathrm{meas}} \,\right|^2 \mathrm{d}\Omega + \underbrace{\alpha(\boldsymbol{\theta}-\boldsymbol{\theta}^{\mathrm{ref}})^T \boldsymbol{D}(\boldsymbol{\theta}-\boldsymbol{\theta}^{\mathrm{ref}})}_{\text{regularization term}}$$

- For hysteretic case (energy based hysteresis model)

$$\boldsymbol{B} = \mu_0(\boldsymbol{H}+\boldsymbol{M}) = \mu_0\Big(\boldsymbol{H} + \sum_{j=1}^{N}\omega_j \boldsymbol{M}_j\Big)$$

$\boldsymbol{M}$ ... magnetization
ω_j ... weighting parameters (parameters to be identified)

we arrive at

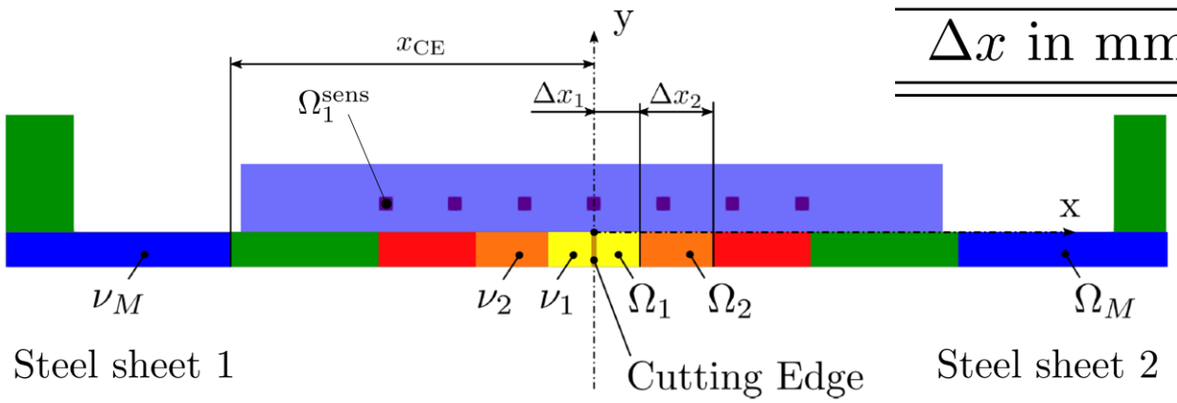
$$\nu = \frac{|\boldsymbol{H}|}{|\boldsymbol{B}|} = \frac{|\boldsymbol{H}|}{\mu_0}\left(|\boldsymbol{H}|^2 + 2\boldsymbol{H}\cdot\sum_{j=1}^{N}\omega_j\boldsymbol{M}_j + |\boldsymbol{M}|^2\right)^{-1/2} \quad ; \qquad \boldsymbol{\theta} = \big(\omega_1, \omega_2, \ldots, \omega_N\big)^T$$

$$\frac{\partial\nu}{\partial\theta_j} = \frac{\partial\nu}{\partial\omega_j} = -\frac{|\boldsymbol{H}|}{\mu_0}\,\frac{\boldsymbol{H}\cdot\boldsymbol{M}_j + \boldsymbol{M}\cdot\boldsymbol{M}_j}{\big(|\boldsymbol{H}|^2 + 2\boldsymbol{H}\cdot\boldsymbol{M} + |\boldsymbol{M}|^2\big)^{3/2}}$$

Numerical Results

Length Δx_l for each subdomain Ω_l

	Ω_1	Ω_2	Ω_3	Ω_4	Ω_5
Δx in mm	0.25	0.5	0.75	1.5	13



Parameters c_0, c_1 and c_2 for initial ν_l^{init}, reference ν_l^{ref} and exact reluctivities ν_l^{exact}

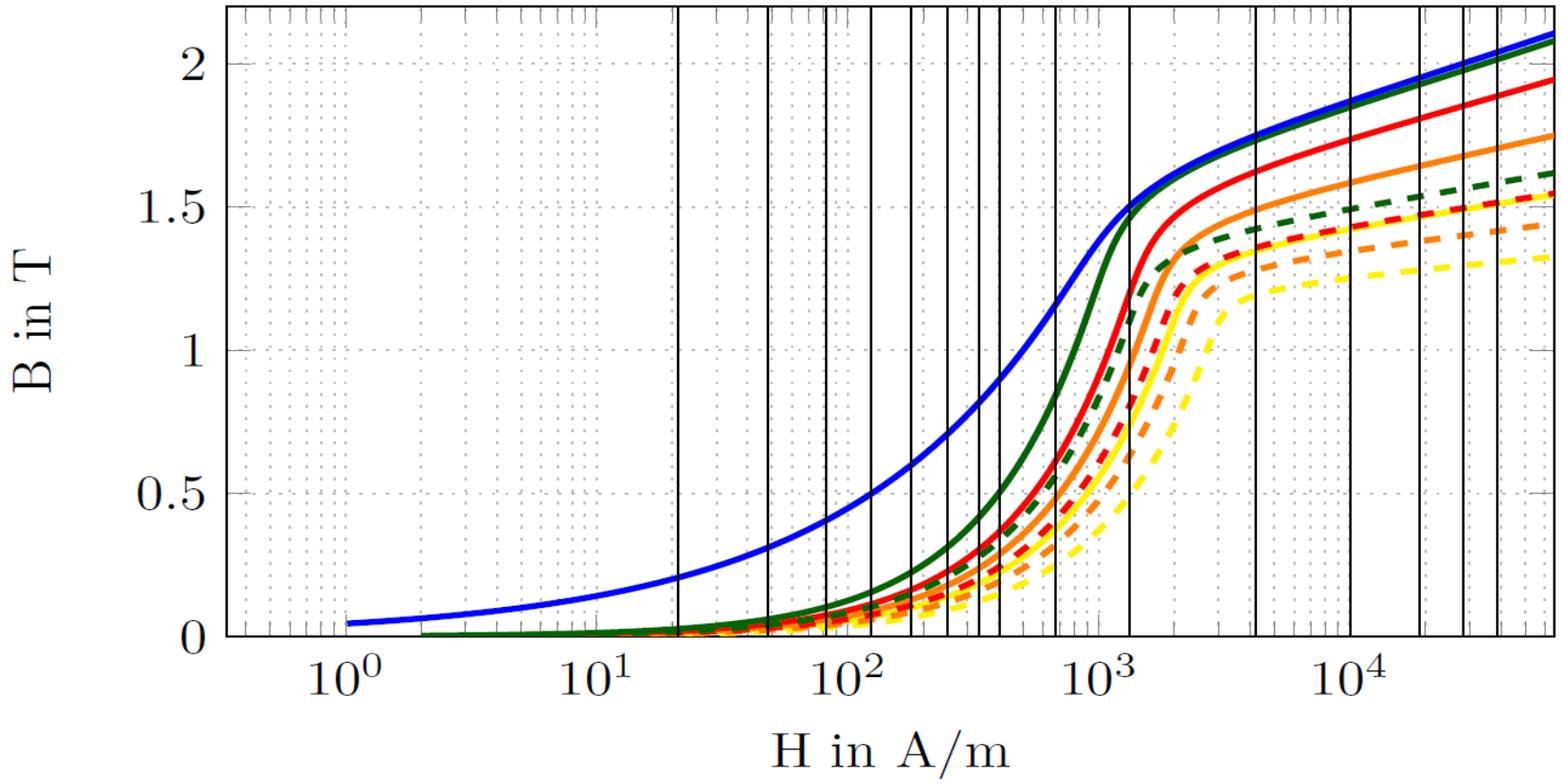
		$\boldsymbol{\theta}^{\mathrm{init}}$			$\boldsymbol{\theta}^{\mathrm{ref}}$			$\boldsymbol{\theta}^{\mathrm{exact}}$		
Domain	Color	c_0	c_1	c_2	c_0	c_1	c_2	c_0	c_1	c_2
Ω_1		2700	1.2	37.5	2250	1.0	31.25	1800	0.8	25
Ω_2		2100	0.75	30.0	1750	0.625	25.0	1400	0.5	20
Ω_3		1650	0.6	25.5	1375	0.5	21.25	1100	0.4	17
Ω_4		1200	0.375	24.0	1000	0.3125	20.0	800	0.25	16
Ω_5		–	–	–	–	–	–	500	0.2	15

Numerical Results

BH-curves for

- Solid line: excate BH curves
- Dashed line: initial BH-curves

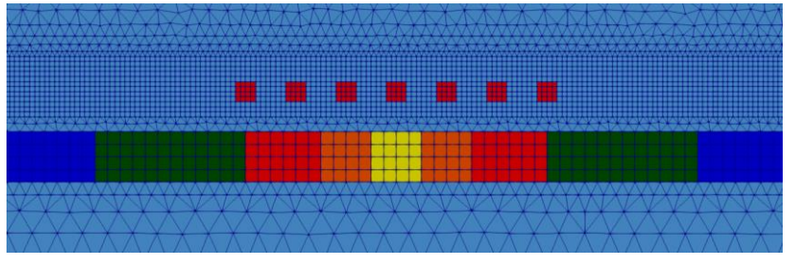
The excitation levels are marked with black solid vertical lines



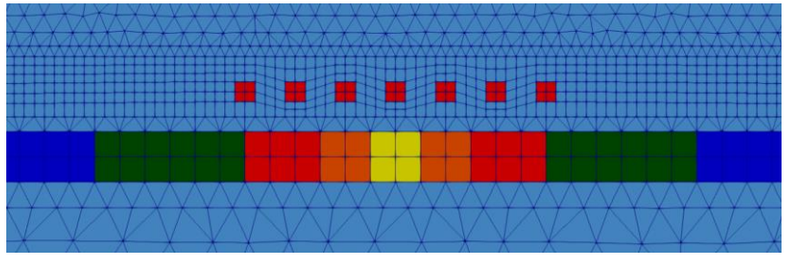
Numerical Results

Zoomed FE model of the sensor-actuator system for computing the measurement data and used for the inverse scheme.

Mesh for computing mesasured data



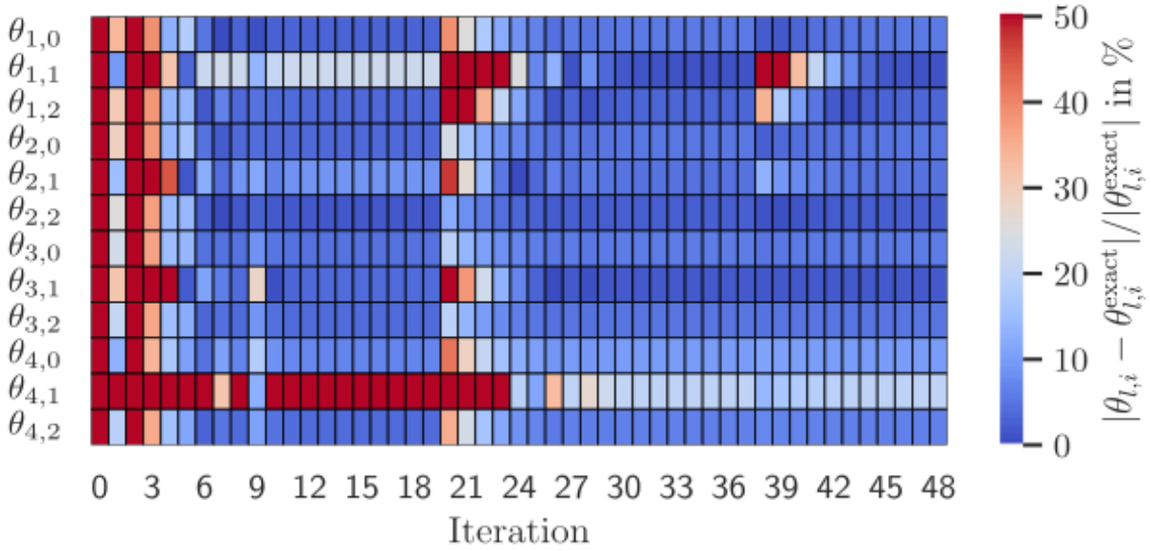
Mesh used for inverse scheme



Convergence behavior of the relative L_2 error norm

Numerical Results

Relative error of the searched for parameters $\boldsymbol{\theta}$

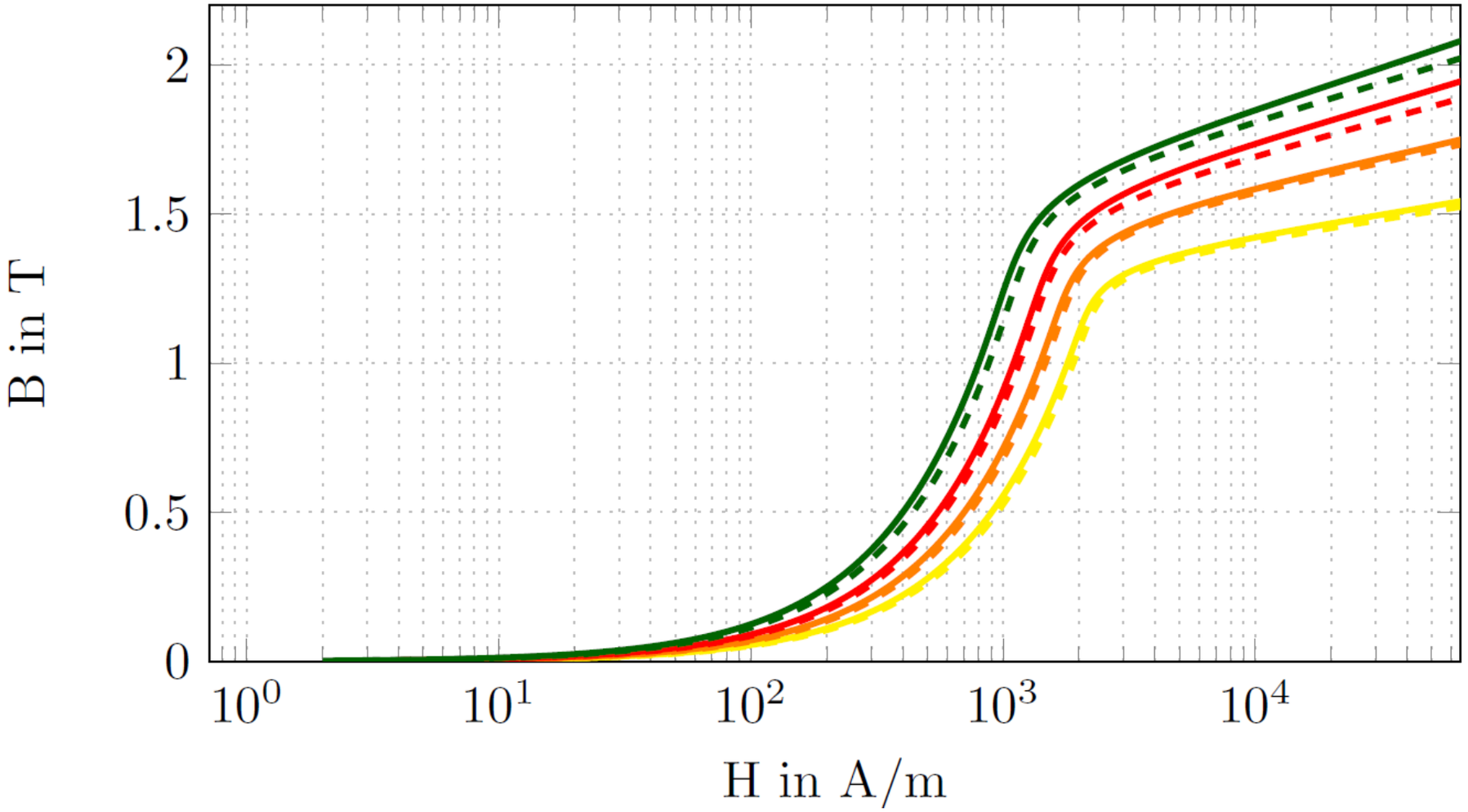


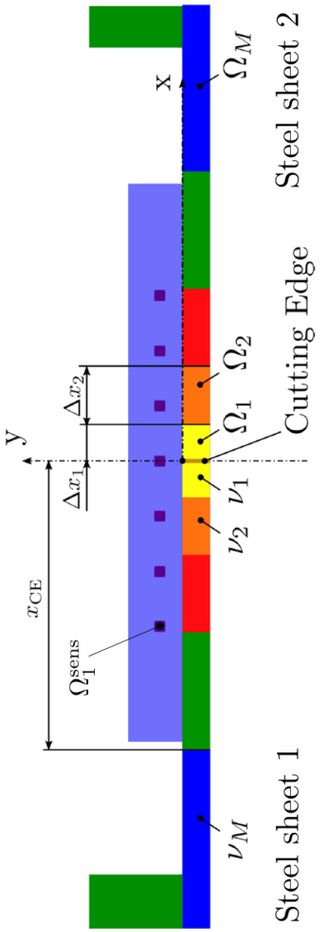
Optimized parameter $\boldsymbol{\theta}^{\text{opt}}$ and relative error $\text{err}_{\boldsymbol{\theta}}^{\text{rel}}$

	$\boldsymbol{\theta}^{\text{opt}}$			$\text{err}_{\boldsymbol{\theta}}^{\text{rel}}$ in %		
Subdomain	c_0	c_1	c_2	c_0	c_1	c_2
Ω_1	1898.4	0.79	25.75	5.47	0.95	2.99
Ω_2	1469.9	0.48	20.44	4.99	4.49	2.17
Ω_3	1160.7	0.41	17.82	5.52	1.70	4.81
Ω_4	876.9	0.20	16.99	9.62	19.73	6.18

Numerical Results

BH-curves for $\boldsymbol{\theta}^{\mathrm{exact}}$ exact (solid line) and $\boldsymbol{\theta}^{\mathrm{opt}}$(dashed line) for the different subdomains.





XML-Syntax

Adjoint PDE

```
<magneticEdgeAdj formulation="A">
   <regionList>
     <region name="V_mu1" matDependIds="rel"/>
     :
   </regionList>
   <matDependencyList>
      <reluctivity id="rel">
         <coupling pdeName="magneticEdge">
            <quantity name="magFluxDensity"/>
         </coupling>
      </reluctivity>
   </matDependencyList>
```

```
<magneticEdgeAdj formulation="A">
   :
   <bcsAndLoads>
      <measuredFluxDensity name="V_sensor_1">
         <fileData file="BmeasSensor1.txt"/>
      </measuredFluxDensity>
      :
   </bcsAndLoads>
```

XML-Syntax

Adjoint PDE

$$RHS = -\sum_{s=1}^{N_s} \frac{1}{|\Omega_s^{\text{sens}}|} \int_{\Omega_s^{\text{sens}}} \left(\boldsymbol{B}_{p,e,s}^{\text{meas}} - \boldsymbol{B}_{p,e,s} \right) \cdot \boldsymbol{\nabla} \times \boldsymbol{v} \, \mathrm{d}\Omega$$

```
<magneticEdgeAdj formulation="A">
    <bcsAndLoads>
        :
        <measuredFluxDensity name="V_sensor_1">
          <fileData file="BmeasSensor1.txt"/>
        </measuredFluxDensity>
        :
        <fluxDensity name="V_sensor_1">
          <coupling pdeName="magneticEdge">
            <quantity name="magFluxDensity" dofs="all"/>
          </coupling>
        </fluxDensity>
        :
```

XML-Syntax

Adjoint PDE

```
<magneticEdgeAdj formulation="A">
  <storeResults>
    <regionResult type="magGradAdjParam1">
      <regionList>
        <region name="V_mu1"/>
        :
    </regionList>
```

$$\left\langle \frac{\partial \mathcal{A}}{\partial \theta_i}(\boldsymbol{u}, \boldsymbol{\theta}), \boldsymbol{w} \right\rangle = \sum_{p=1}^{N_\mathrm{p}} \sum_{e=1}^{N_\mathrm{e}} \int_{\Omega} \frac{\partial \nu(B_{p,e})}{\partial \theta_i} \, \boldsymbol{\nabla} \times \boldsymbol{u}_{p,e} \cdot \boldsymbol{\nabla} \times \boldsymbol{w}_{p,e} \, \mathrm{d}\Omega'$$

Coupling between PDE and adjoint PDE

```
<couplingList>
  <iterative PDEorder="magneticEdge; magneticEdgeAdj">
    <convergence logging="yes" maxNumIters="1" stopOnDivergence="no">
      <quantity name="magPotential" value="1"/>
    </convergence>
  </iterative>
</couplingList>verbatim}
```

Implementation

Domain.cc:

- Domain::CreateSinglePDEs

```
else if (actPdeName == "magneticEdgeAdj") {
      ptSinglePde_[i] = new MagEdgeAdjPDE(defaultGrid, actPdeNode, infoNode,
                                          simState_, this);
```

MagEdgeAdjPDE.cc

- MagEdgeAdjPDE::MagEdgeAdjPDE

```
pdename_           = "magneticEdgeAdj";
pdematerialclass_ = ELECTROMAGNETIC;
formulation_ = MagBasePDE::EDGE;
```

Implementation

MagEdgeAdjPDE.cc

- MagEdgeAdjPDE::DefineStandardIntegrators()

```
if ( matDepenTypes.Find(RELUCTIVITY) != -1 ) {
  :
  ReadMaterialDependency( "reluctivity", dispDofNames, ResultInfo::VECTOR, isComplex_,
                          ent, coef, updatedGeo_ );
  //coef-Fnc for magnetic reluctivity
  curCoef = actMat->GetScalCoefFncNonLin( MAG_RELUCTIVITY_SCALAR, Global::REAL, coef);
  :
  //second part according to newton scheme (tangenial part)
  nuDeriv = actMat->GetTensorCoefFncNonLin( MAG_RELUCTIVITY_DERIV, FULL, Global::REAL,
                                            coef );
  :
  //coeff-Function for derivations of nu w.r.t. parameters
  PtrCoefFct derivParam1 = actMat->GetScalCoefFncNonLinDerivParam(
                          MAG_RELUCTIVITY_DERIV_P1, Global::REAL, coef);
  nuDerivParamM1_[actRegion] = derivParam1;
  :
}
```

Implementation

MagneticScalarPotentialAdjPDE.cc (for hysteresis operator)

- MagneticScalarPotentialAdjPDE::DefineIntegrators()

```
if ( matDepenTypes.Find(PERMEABILITY) != -1 ) {
   :
   //get the material model (hysteresis): in matModelCoef_
   domain_->SetRegion4Hyst(actRegion);
   mu = iterCplPde_->GetCouplingCoefFct(MAG_MAGNETIZATION, actSDList,
                                        "magneticScalarPotential", updatedGeo_);

   nlFluxCoefm_[actRegion].reset(new CoefFunctionMulti(CoefFunction::VECTOR,
                                       dim_, 1, isComplex_, true));
   nlFluxCoefm_[actRegion]->AddRegion(actRegion, mu);
   domain->SetRegion4Hyst(NO_REGION_ID);
   :
```

Thank you

for your attention!